



# Calculation Policy

### Aims of the Calculation Policy

- Outline appropriate methods to use to teach clarity and understanding in addition, subtraction, multiplication and division across the primary phase
  - Clarify progression in mathematics across Key Stage 1 and Key Stage 2
- Ensure that children learn with understanding using the **concrete, pictorial and abstract** methods shown, and not just through procedural methods to simply be remembered
  - Enable pupils to show their understanding using a variety of methods and explain the reasoning behind their findings

### The Aims of the Curriculum

The national curriculum for mathematics aims to ensure that all pupils:

- Become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problem problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- Reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- Can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering solutions

### Concrete – Pictorial – Abstract

**Concrete** – using manipulatives

**Pictorial** – drawing their own representations of the concrete e.g. number lines, base ten, jottings

**Abstract** – calculations using numerals and symbols

*“Children aged seven to ten years old work in primarily concrete ways and that the abstract notions of mathematics may only be accessible to them through embodiment in practical resources.” Jean Piaget (1951)*

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part – whole model Starting at the bigger number and counting on Regrouping to make 10	Add a two digit number and ones. (Add TO + O) Add two, two digit numbers (TO + TO) Add a two digit number and tens Add three one digit numbers Using the inverse to find the missing number (Difference)	Column addition with no regrouping. (Numbers up to three digits) Column addition with regrouping. (Numbers up to three digits)	Column method regrouping (up to 4 digits)	Column method regrouping with at least 4 digits, including money and measures	Column method regrouping with several numbers (increasing complexity), including money, measure and decimals with different decimal points.
Subtraction	Taking away ones Counting back Find the difference Part-Part-Whole model Make 10	Regroup a ten into ten ones. Partitioning to subtract without regrouping Make ten strategy	Column subtraction without regrouping Column subtraction with regrouping Introduce decimal subtraction through context of money.	Column subtraction subtract with up to 4 digits (with and without regrouping)	Column subtraction Subtract with at least 4 digits, including money and measures.	Column Subtraction Subtract with increasingly large and more complex numbers and decimal values.
Multiplication	Doubling Counting in multiples /repeating groups Repeated addition Understanding arrays	Arrays showing commutivity Doubling Counting in multiples of 2, 3, 4, 5, 10 from 0 Using the inverse	Grid method Expanded short multiplication (TO x O) leading to introduction of short multiplication (TO x O)	Expanded short multiplication (HTO x O) leading to short multiplication (HTO x O)	Column Multiplication Up to 4 digits by a 1 or 2 digit number ThHTO x O ThHTO x TO Long multiplication for two-digit numbers	Column Multiplication Up to 4 digits by a 1 or 2 digit number ThHTO x TO Long multiplication for two-digit numbers
Division	Sharing objects into groups Division as grouping	Division as sharing and grouping	Division with arrays Division using remainders Introducing Short division (no remainders, no regrouping) Introducing Short division (no remainders, regrouping)	Short division Divide at least 3 digit numbers by 1 digit. (no regrouping)  Short division Divide at least 3 digit numbers by 1 digit. (regrouping)	Short Division: Divide numbers up to 4 digits by a 1 digit number (regrouping)	Short Division: Divide numbers up to 4 digits by a 1 digit number (regrouping)  Divide numbers up to 4 digits by a two-digit number using the formal written method of short division  Long division



# Mathematical Vocabulary

Correct terminology	Incorrect Terminology
ones	units
Is equal to (is the same as)	equals
zero	Oh (the letter o)
exchange exchanging regrouping	Stealing borrowing
sharing grouping groups	
calculation equation	Generic term of 'sum' or 'number sentence'
bar model	
known unknown	
whole part	
Key Vocabulary	
<p>Addition:</p> $\begin{array}{c} \text{Addend} \quad 8 + 3 = 11 \quad \text{Sum} \\ \text{Addend} \end{array}$	$\begin{array}{r} 25 \text{ multiplicand} \\ \times 6 \text{ multiplier} \\ \hline 150 \text{ product} \end{array} \quad \left. \vphantom{\begin{array}{r} 25 \\ \times 6 \end{array}} \right\} \text{factors}$
$\begin{array}{c} \text{Minuend} \quad 8 - 3 = 5 \quad \text{Difference} \\ \text{Subtrahend} \end{array}$	$\begin{array}{r} \text{Dividend} \quad 40 \div 8 = 5 \quad \text{Quotient} \\ \text{Divisor} \end{array} \quad \begin{array}{r} 6 \text{ --- quotient} \\ 4 \overline{) 24} \text{ --- dividend} \\ \text{divisor} \end{array}$

## Definitions

**Addend:** Any of the numbers that are added together.

**Sum:** The result of adding two or more numbers

**Minuend:** The first number in a subtraction. The number from which another number is subtracted.

**Subtrahend:** The number that is to be subtracted. The second number in a subtraction.

**Difference:** The result of subtracting one number from another. How much one number differs from another.

**Division:** Division is splitting into equal parts or **groups**.

**Dividend:** The amount that you want to divide up.

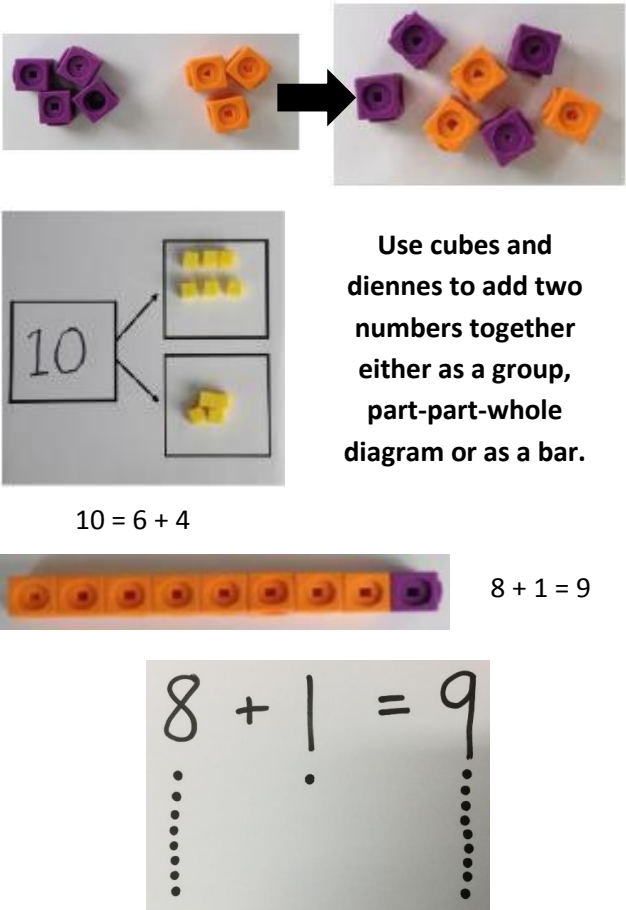
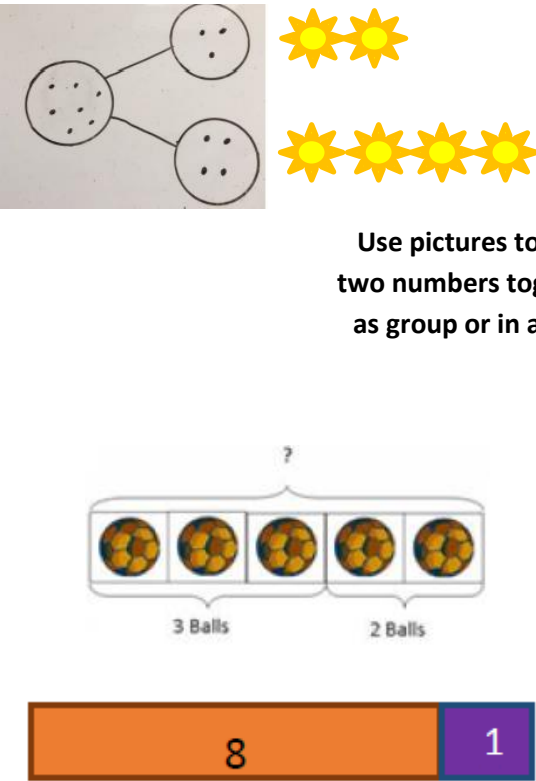
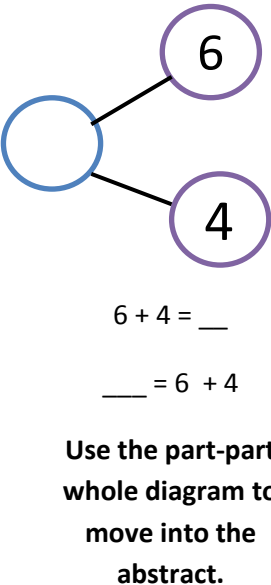
**Divisor:** The number you divide by.

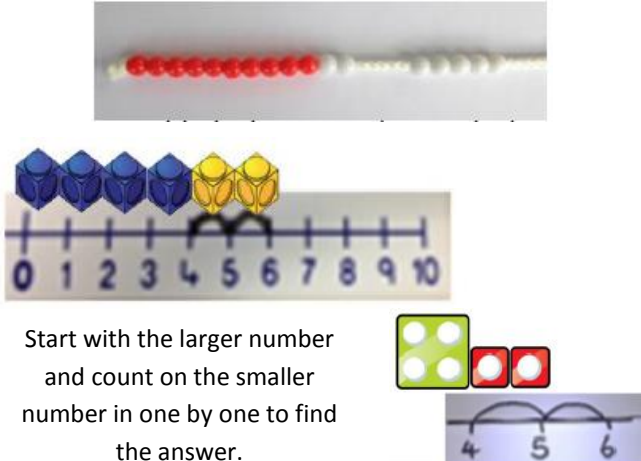
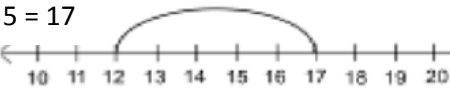
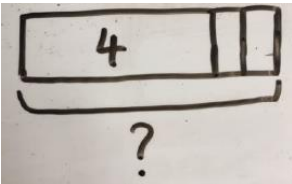
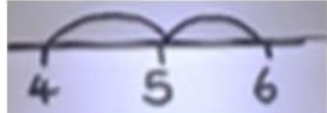

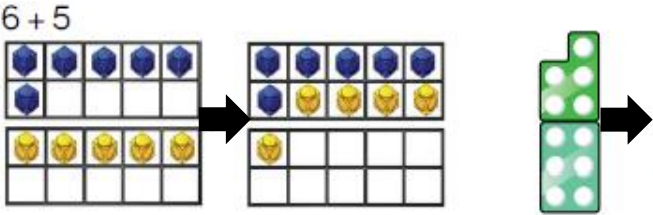
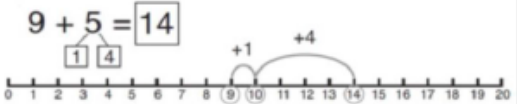
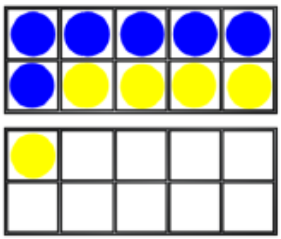
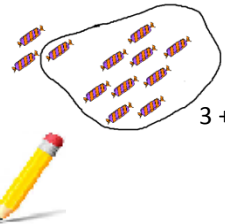
**Quotient:** The answer after you divide one number by another.

**Multiplicand:** The number that gets multiplied/ the size of the group.


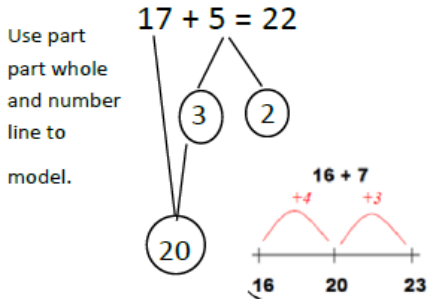
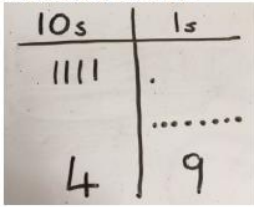
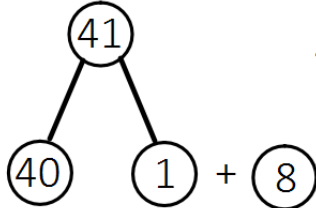
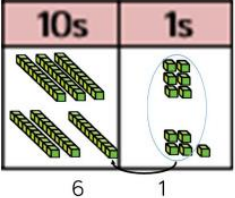
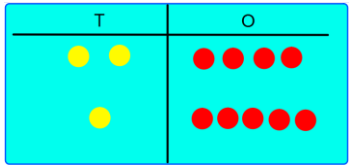
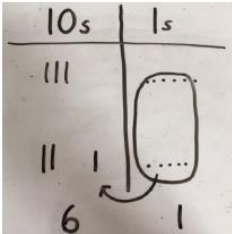
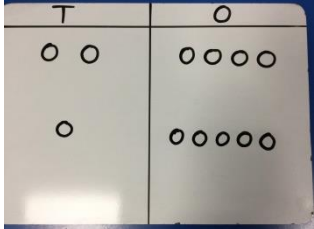
**Multiplier:** The number that you are multiplying by/ the number of groups.

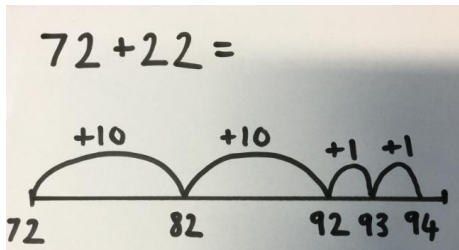

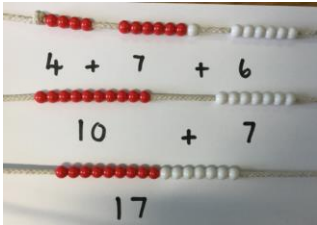
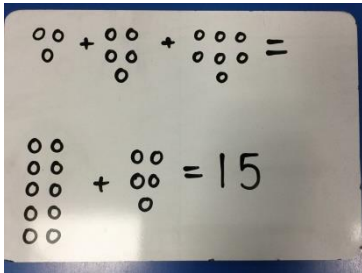
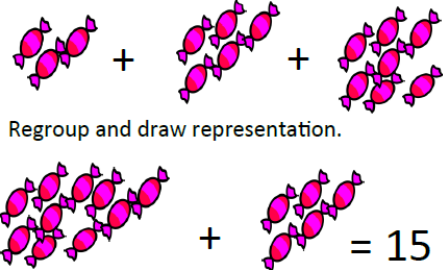
**Product:** The answer when two or more numbers are multiplied together.

Addition Year 1	Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole: part – whole model</p>	 <p>Use cubes and diennes to add two numbers together either as a group, part-part-whole diagram or as a bar.</p> <p><math>10 = 6 + 4</math></p> <p><math>8 + 1 = 9</math></p> <p><math>8 + 1 = 9</math></p>	 <p>Use pictures to add two numbers together as group or in a bar.</p>	 <p><math>6 + 4 = \underline{\quad}</math></p> <p><math>\underline{\quad} = 6 + 4</math></p> <p>Use the part-part whole diagram to move into the abstract.</p>

<p>Starting at the bigger number and counting on</p>	 <p>Start with the larger number and count on the smaller number in one by one to find the answer.</p>	<p><math>12 + 5 = 17</math></p>  <p><b>Start at the larger number on the number and count on in ones or in one jump to find the answer.</b></p>  <p><b>A bar model which encourages children to count on rather than count all.</b></p>	<p><math>12 + 5 = 17</math></p> <p>Place the larger number in your head and count on the smaller number to find your answer.</p> <p>What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? <math>4 + 2</math></p>  <p>Use a blank number line to count on.</p>
<p>Regrouping to make 10</p> <p>This is an essential skill for column addition later.</p>	 <p><math>9 + 3 = 12</math></p> <p>Start with the bigger number and use the smaller number to make 10.</p> <p>Use ten frames, bead strings counters/cubes or Numicon.</p> 	<p><math>9 + 5 = 14</math></p>  <p>Use pictures or a number line.</p> <p>Regroup or partition the smaller number to make 10</p>   <p><math>3 + 9 = 12</math></p> <p>Children to fill in a ten frame with pencils, counters or cubes.</p>	<p><math>7 + 4 = 11</math></p> <p>If I am seven how many more do I need to make 10? How many more many more do I add on now?</p> <p>Children to develop an understanding of equality e.g.</p> <p><math>6 + \square = 11</math>  <math>6 + 5 = 5 + \square</math>  <math>6 + 5 = \square + 4</math></p>

# YEAR 2 ADDITION

Addition Year 2	Concrete	Pictorial	Abstract
<p>Add a two digit number and ones. Add TO + O</p>	<p>Continue to develop understanding of partitioning and place value.</p> <p><math>41 + 8</math></p> 	<p>Use part part whole and number line to model.</p> <p><math>17 + 5 = 22</math></p>  <p>Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.</p> 	<p><math>41 + 8 =</math></p>  <p><math>1 + 8 = 9</math> <math>40 + 9 = 49</math></p> <p><math>(40) + (9) = 49</math></p>
<p>Add two, two digit numbers TO + TO</p>	<p><math>36 + 25 =</math></p>  <p>TO + TO using base 10 first before moving onto place value counters. Continue to develop understanding of partitioning and place value.</p>  <p><math>24 + 15 =</math></p>	 <p>After practically using the resources, children to represent the base 10 in a place value chart.</p> <p><math>36 + 25 =</math></p>  <p>Children can draw the counters in a place grid to help them solve additions.</p>	<p><math>36 + 25 =</math></p> <p>Children to apply their understanding from their use of concrete and pictorial methods to answer questions using a formal column written method.</p> $\begin{array}{r} 36 \\ 25 + \\ \hline 61 \\ \hline 1 \end{array}$

		Use a number line to support bridging ten. 	$\begin{array}{r} 35 + 25 = \\ 30 + 6 \quad 20 + 5 \\ \hline 50 + 11 = 61 \end{array}$
Add a two digit number and tens	 $25 + 10 = 35$ Explore that the ones digit does not change	$\begin{array}{r} 27 + 30 \\ +10 \quad +10 \quad +10 \\ \hline 27 \quad 37 \quad 47 \quad 57 \end{array}$	$\begin{array}{l} 27 + 10 = 37 \\ 27 + 20 = 47 \\ 27 + \square = 57 \end{array}$
Add three one digit numbers	 $4 + 7 + 6 = 17$ Put the 4 and 6 together to make 10. Add on the 7.	  Regroup and draw representation.	$\begin{array}{r} (4) + 7 + (6) = \boxed{10} + \boxed{7} \\ 10 \\ = \boxed{17} \end{array}$ Combine the two numbers that make/bridge ten then add on the third.

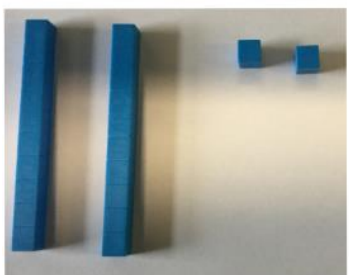
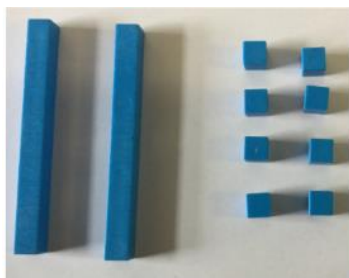


Using the  
inverse to find  
the missing  
number.

Difference

$$6 + \square = 28$$

$$\square = 28 - 6$$



$$6 + \square = 28$$

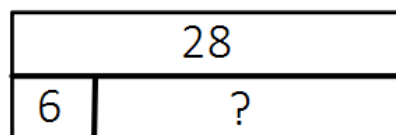
$$6 + \square = 28$$

$$\square = 28 - 6$$

$$6 + \square = 28$$

$$\square = 28 - 6$$

Bar Model:

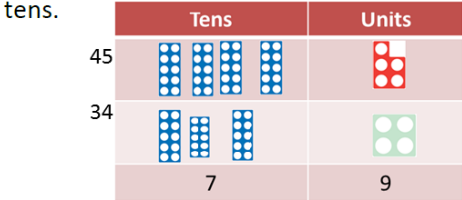
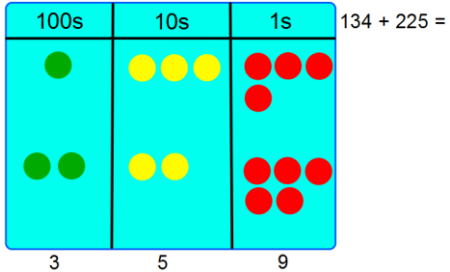
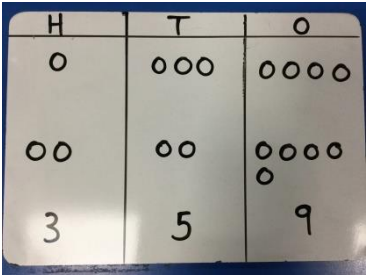
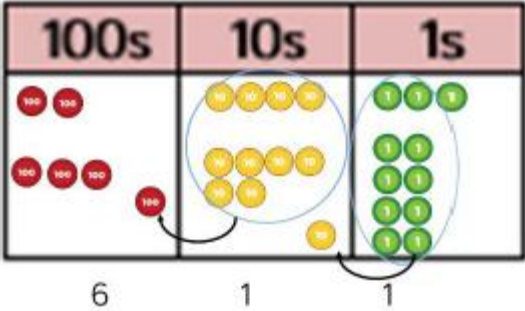
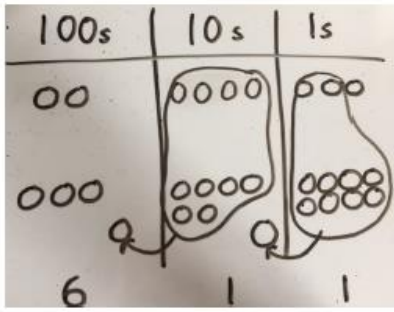


$$\square = 28 - 6$$

$$6 + 22 = 28$$

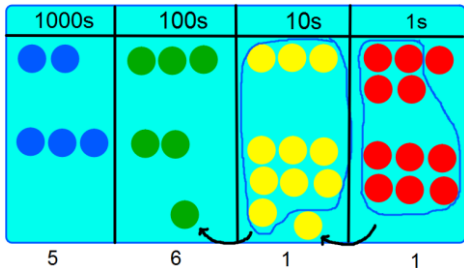
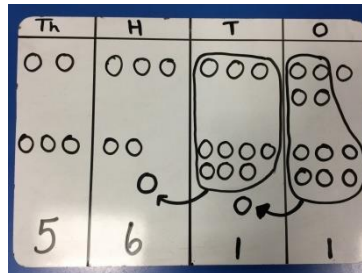
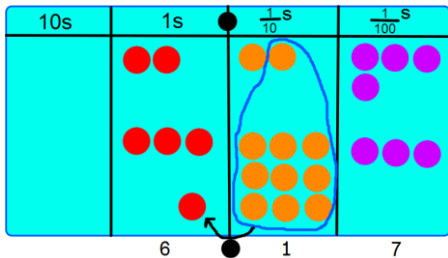
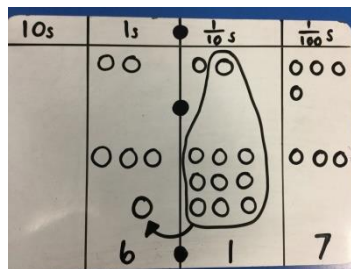
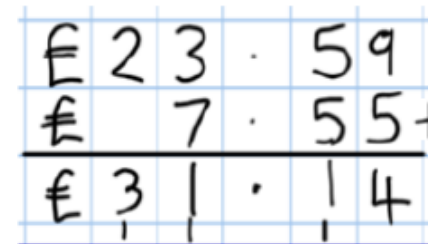
$$22 = 28 - 6$$

# YEAR 3 ADDITION

Addition Year 3	Concrete	Pictorial	Abstract
<p>Column addition with no regrouping</p> <p>Numbers up to three digits.</p>	<p>Add together the ones first, then the tens.</p>   <p>134 + 225 =</p>		$\begin{array}{r} 134 \\ 225 + \\ \hline 359 \end{array}$
<p>Column addition with regrouping</p> <p>Numbers up to three digits.</p>	<p>When there are 10 ones in the 1s column we regroup for 1 ten, when there are 10 tens in the 10s column we regroup for 1 hundred.</p> 	<p>Children to represent the counters in a place value chart circling when they regroup.</p> 	$\begin{array}{r} 243 \\ 368 + \\ \hline 611 \\ 1 \quad 1 \end{array}$

			<p><u>Measure:</u></p> $  \begin{array}{r}  £53.26 \\  £8.58 + \\  \hline  £61.84 \\  \hline  \end{array}  $ <p style="text-align: center;">1      1</p>
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# YEAR 4 YEAR 5 YEAR 6 ADDITION

Addition Year 4, 5 and 6	Concrete	Pictorial	Abstract
<p>Year 4: Column method regrouping (up to 4 digits)</p>	<p>When there are 10 ones in the 1s column we regroup for 1 ten, when there are 10 tens in the 10s column we regroup for 1 hundred. When there are 10 hundreds in the 100s column we regroup for 1 thousand.</p> 		$\begin{array}{r} 2335 \\ 3276 + \\ \hline 5611 \\ 11 \end{array}$
<p>Year 5: Column method regrouping (with more than 4 digits)</p> <p>Add decimals with 2 decimal places, including money</p>	<p>As year 4, including numbers with more than 4 digits.</p>  <p>When there are 10 tenths in the 1/10s column we regroup 10 tenths for 1 one, when there are 10 tens in the 10s column we regroup for 1 hundred. When there are 10 hundreds in the 100s column we regroup for 1 thousand.</p>		$\begin{array}{r} 2.24 \\ 3.93 + \\ \hline 6.17 \\ 1 \end{array}$ 



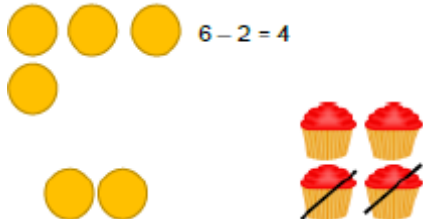
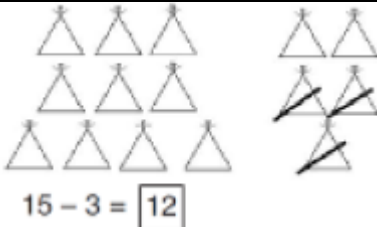
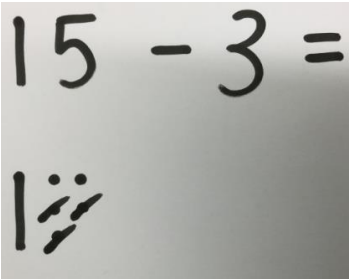

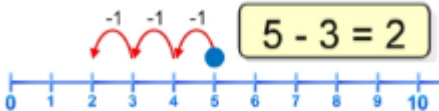
<p>Year 6: Add several numbers with increasing complexity.</p> <p>Including adding money, measure and decimals with different decimal points.</p>	<p>Use place value counters as above if still necessary.</p>	<p>Use jottings as above if still necessary.</p>	<div><div><div>81059</div><div>20551</div><div>5301</div><div>3668</div><div>+</div><div><div>110579</div><div>1111</div></div></div><div><div>Insert zeros for place holders.</div><div><div>23.361</div><div>9.080</div><div>59.770</div><div>+ 1.300</div><div><div>93.511</div><div>21.2</div></div></div></div></div>
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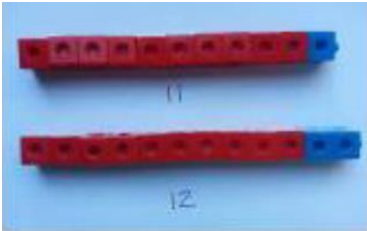
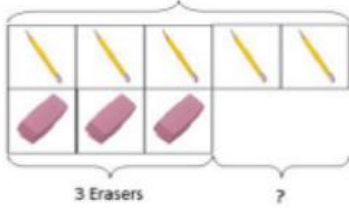
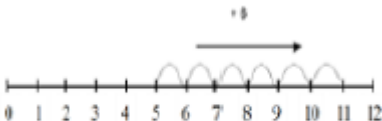
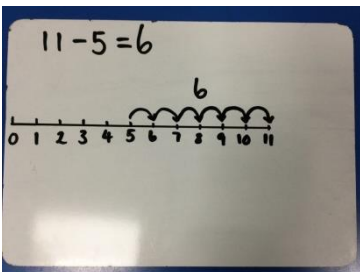
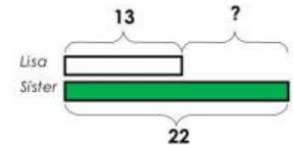
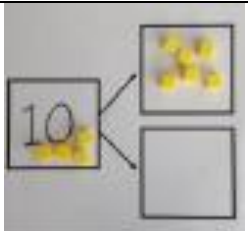
## Conceptual variation; different ways to ask children to solve $21 + 34$

<div><div><div><div><div></div><div>?</div></div><div><div>21</div><div>34</div></div></div></div><div><table><tr><td colspan="2">?</td></tr><tr><td>21</td><td>34</td></tr></table></div></div>	?		21	34	<div>Word problems: In year 3, there are 21 children and in year 4, there are 34 children. How many children in total?</div> <div>21 + 34 = 55. Prove it</div>	<div><div><div>21</div><div>+34</div><div></div></div><div>21 + 34 =</div><div><div></div><div>= 21 + 34</div></div><div>Calculate the sum of twenty-one and thirty-four.</div></div>	<div><div><div><div><div></div><div></div></div><div><div></div><div></div></div></div><div>+</div><div><div><div><div></div><div></div></div><div><div></div><div></div></div></div><div><div></div><div></div></div></div></div><div>Missing digit problems:</div><div><table><tr><th>10s</th><th>1s</th></tr><tr><td><div><div>10</div><div>10</div></div></td><td><div>1</div></td></tr><tr><td><div><div>10</div><div>10</div><div>10</div></div></td><td>?</td></tr><tr><td>?</td><td>5</td></tr></table></div></div>	10s	1s	<div><div>10</div><div>10</div></div>	<div>1</div>	<div><div>10</div><div>10</div><div>10</div></div>	?	?	5
?															
21	34														
10s	1s														
<div><div>10</div><div>10</div></div>	<div>1</div>														
<div><div>10</div><div>10</div><div>10</div></div>	?														
?	5														

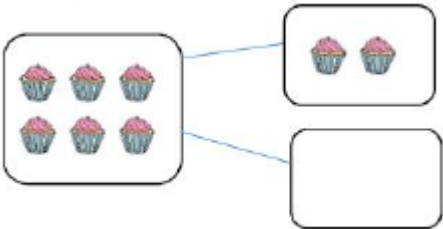


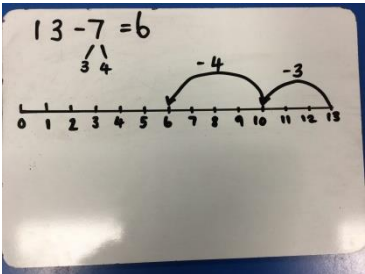
Subtraction	Concrete	Pictorial	Abstract
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# YEAR 1 SUBTRACTION

Subtraction Year 1	Concrete	Pictorial	Abstract
Taking away ones.	<p>Use physical objects, counters, diennes, cubes etc. to show how objects can be taken away.</p>  $6 - 2 = 4$	 <p>15 - 3 = 12</p> <p>Cross out drawn objects to show what has been taken away.</p> 	$7 - 4 = 3$  $16 - 9 = 7$
Counting back.	 <p>Move the beads along the bead string as you count backwards in ones.</p> <p>Move objects away from the group, counting backwards</p>	 <p>5 - 3 = 2</p> <p>Count back in ones using a number line.</p>	<p>Put 13 in your head, count back 4. What number are you at?</p>

<p>Find the difference</p>	<p>Compare objects and amounts.</p>  <p>Use cubes to build towers or make bars to find the difference.</p>  <p>Use basic bar models with items to find the difference.</p>	<p>Count on using a number line to find the difference.</p> <p>+6</p>   <p><b>Comparison Bar Models</b></p> <p>Draw bars to find the difference between 2 numbers.</p> <p>Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.</p> 	<p>Hannah has 12 grapes and her sister has 5. How many more does Hannah have than her sister?</p> <p>Helen has 11 plums and her sister has 3. Find the difference between the number of plums.</p>
<p>Part Part Whole model.</p>	 <p>Link to addition. Use PPW model to model the inverse.</p>	<p>Use pictorial representations to show the part.</p>	<p>Move to using numbers within the part part whole model.</p>



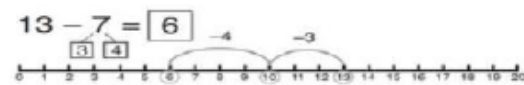
	<p>If 10 is the whole and 6 is one of the parts, what is the other number?  <math>10 - 6 = 4</math></p>		
Make 10.	 <p><math>14 - 9 =</math></p> <p>Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.</p>	<p>Jump back 3 first, then another 4. Use ten as the stopping point.</p> <p>Children must be encouraged to draw a number line and be able to interpret one.</p> 	<p><math>16 - 8</math></p> <p>How many do we take off first to get to 10?          How many left to take off?</p>

### Bar Model

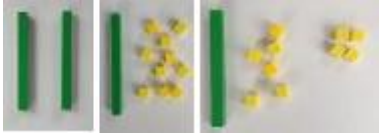
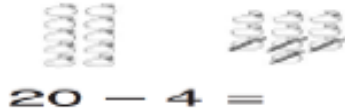
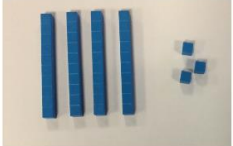
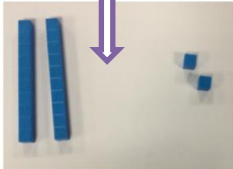


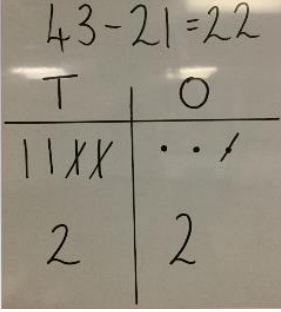
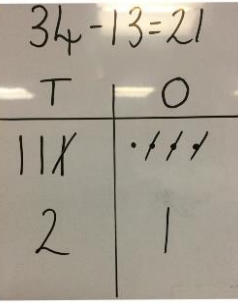
$$28 - 6 = \square$$

28	
6	?

T	O
00	00
	<del>00</del>
	<del>00</del>
	<del>00</del>



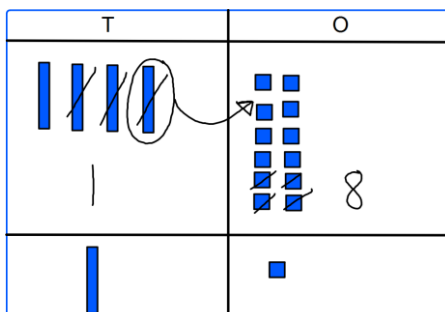
# YEAR 2 SUBTRACTION

Subtraction Year 2	Concrete	Pictorial	Abstract
Regroup a ten into ten ones.  $20 - 4 = 16$  Change a ten into ten ones.	  $20 - 4 = 16$  Change a ten into ten ones.	  $20 - 4 =$	$20 - 4 = 16$
Subtracting without regrouping.  $34 - 13 = 21$	   $34 - 13 = 21$  	 $43 - 21 = 22$  Children draw representations of Dienes and cross off.   	Children to be supported in their understanding by using a formal column method.  $43 - 21 = 22$ $34 - 13 = 21$  $\begin{array}{r} 43 \\ - 21 \\ \hline 22 \end{array}$ $\begin{array}{r} 34 \\ - 13 \\ \hline 21 \end{array}$

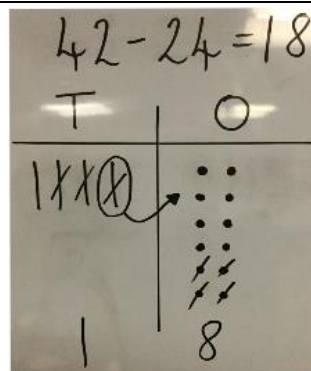
Subtracting  
TO –TO with  
regrouping

$$42 - 24 = 18$$

Start with the ones, can I take away 2 from 2 ones easily? No, I need to exchange one of my tens for ten ones.



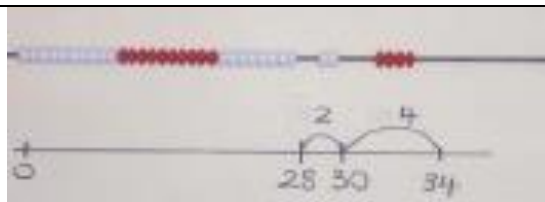
Now I can subtract my ones (remove/cross out cubes from the place value mat).  
Now look at the tens, can I take away 2 tens from the 3 tens easily? Yes.



$$\begin{array}{r} 3 \cancel{4} 2 \\ - 24 \\ \hline 18 \end{array}$$

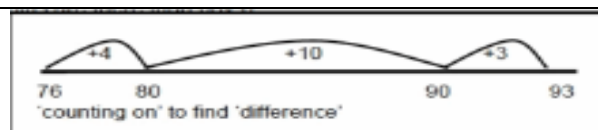
Make ten  
strategy.

Progression  
should be  
crossing one  
ten, crossing  
more than one  
ten, crossing  
the hundreds.

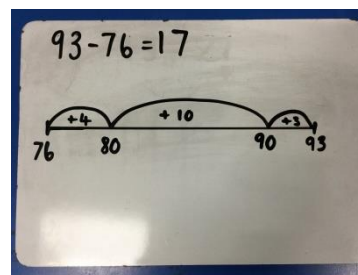


$$34 - 28 =$$

Use a bead bar or bead strings to model counting to next ten and the rest.



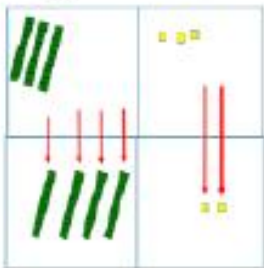
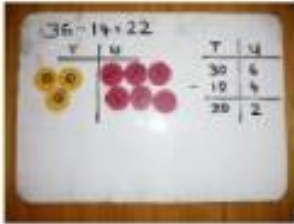

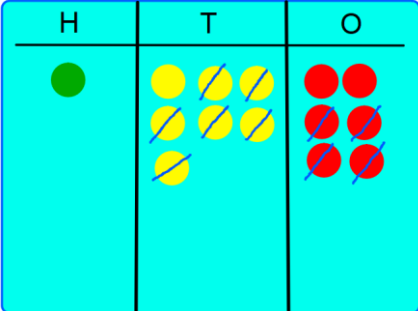
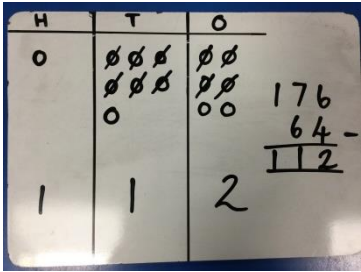
Use a number line to count on to the next ten and then the rest.



$$93 - 76 =$$



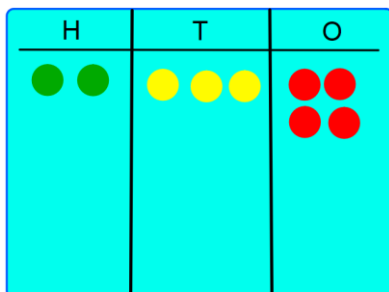
		<p><u>Bar Model:</u></p> <p>93- 76 =</p> <table><tr><td colspan="2">93</td></tr><tr><td>76</td><td>?</td></tr></table>	93		76	?	
93							
76	?						

Subtraction Year 3	Concrete	Pictorial	Abstract
Column subtraction without regrouping.	<p><math>75 - 42 =</math></p>  <p>Use Base 10 to make the bigger number then take the smaller number away.</p>  <p>Show how you partition numbers to subtract. Again make the larger number first.</p>	 <p>Calculations</p> $\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$  <p><math>176 - 64 =</math></p> $\begin{array}{r} 176 \\ - 64 \\ \hline 112 \end{array}$ <p>Draw the base 10 or place value counters alongside the written calculation to help show working.</p>  <p>Children could rub out jottings instead of crossing them out.</p>	$\begin{array}{r} 32 \\ 12 - \\ \hline 20 \end{array}$ $\begin{array}{r} 345 \\ 123 - \\ \hline 222 \end{array}$

Column subtraction with regrouping.

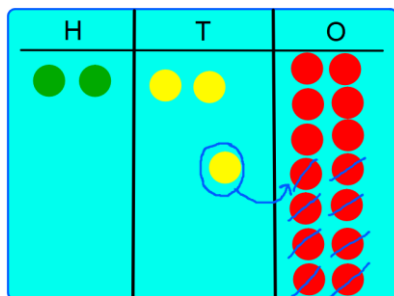
Use base 10 to start with before moving onto place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

Make the larger number with the place value  
counters.



$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

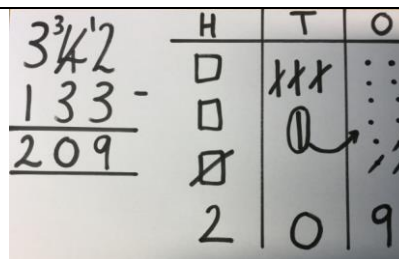
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



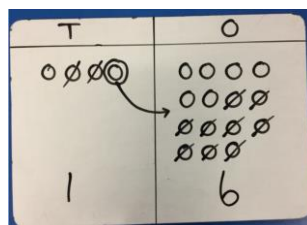
$$\begin{array}{r} 234 \\ - 88 \\ \hline 146 \end{array}$$

Now I can subtract my ones (remove counters from the place value mat).

Now look at the tens, can I take away 8 from the tens easily? I need to exchange one hundred for ten tens.



Children may draw base ten or PV counters and cross off.



Draw the counters onto a place value grid and show what you have taken away by crossing out or rubbing out as well as clearly showing the exchanges you make.

### Column Subtraction

$$\begin{array}{r} 71 \\ 836 \\ 254 \cdot \\ \hline 582 \end{array}$$

Measure:

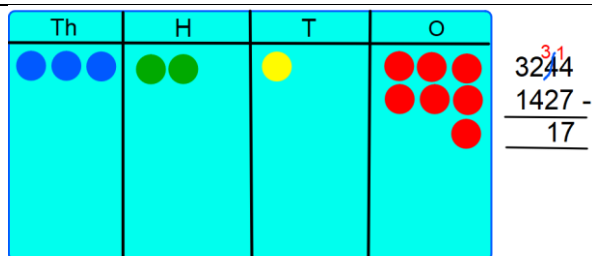
$$\begin{array}{r} \text{£ } 42.34 \\ \text{£ } 6.16 \\ \hline \text{£ } 36.18 \end{array}$$



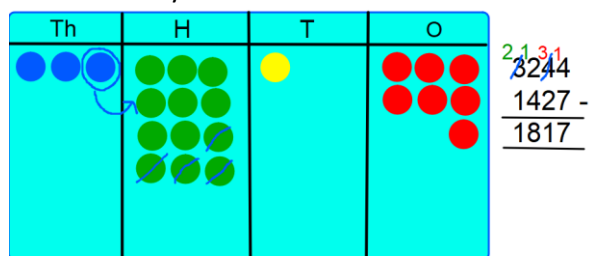


# YEAR 4 YEAR 5 YEAR 6 SUBTRACTION

Subtraction Year 4, 5 and 6	Concrete	Pictorial	Abstract																
<p>Year 4 to subtract with up to 4 digits.</p> <p>Introduce decimal subtraction through context of money.</p>	<div> <table border="1"> <thead> <tr> <th>Th</th><th>H</th><th>T</th><th>O</th></tr> </thead> <tbody> <tr> <td>●●●●</td><td>●●</td><td>●●●●</td><td>●●●●</td></tr> </tbody> </table> <div> <math display="block">\begin{array}{r} 3244 \\ 1427 - \\ \hline \end{array}</math> </div> </div> <p>Start with the ones, can I take away 7 from 4 ones easily? I need to exchange one of my tens for ten ones.</p> <div> <table border="1"> <thead> <tr> <th>Th</th><th>H</th><th>T</th><th>O</th></tr> </thead> <tbody> <tr> <td>●●●●</td><td>●●</td><td>●●●●</td><td>●●●●</td></tr> </tbody> </table> <div> <math display="block">\begin{array}{r} 3244 \\ 1427 - \\ \hline 7 \end{array}</math> </div> </div> <p>Now I can subtract my ones (remove counters from the place value mat).</p> <p>Now look at the tens, can I take away 2 tens from the 3 tens easily? Yes.</p>	Th	H	T	O	●●●●	●●	●●●●	●●●●	Th	H	T	O	●●●●	●●	●●●●	●●●●		<div> <math display="block">\begin{array}{r} 2754 \\ 1562 - \\ \hline 1192 \end{array}</math> </div> <div> <math display="block">\begin{array}{r} £42.34 \\ £6.16 - \\ \hline £36.18 \end{array}</math> </div>
Th	H	T	O																
●●●●	●●	●●●●	●●●●																
Th	H	T	O																
●●●●	●●	●●●●	●●●●																



Now look at the hundreds, can I take away 4 hundred from 2 hundred? I need to exchange one of my thousands for 10 hundreds.



Now I can complete my calculation.

Year 5 –  
Subtract with  
at least 4  
digits,  
including  
money and  
measures.

See Year 4.

See Year 4.

$$\begin{array}{r} 28'00'00'00' \\ 2128 - \\ \hline 28,928 \end{array}$$

Use zeros  
for place-  
holders.

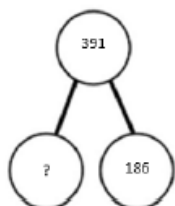
$$\begin{array}{r} 7'00'00'00' \\ 372.5 - \\ \hline 6796.5 \end{array}$$

Year 6 –  
Subtract with  
increasingly  
large and  
more complex  
numbers and  
decimal  
values.

$$\begin{array}{r} \cancel{1} \cancel{8} \cancel{0}, 699 \\ - 89,949 \\ \hline 60,750 \end{array}$$
  

$$\begin{array}{r} \cancel{1} \cancel{0} 5 \cdot \cancel{4} 19 \\ - 36 \cdot 08 \text{ kg} \\ \hline 69 \cdot 339 \text{ kg} \end{array}$$

## Conceptual variation; different ways to ask children to solve $391 - 186$



391	
186	?

Raj spent £391, Timmy spent £186.  
How much more did Raj spend?

Calculate the difference between 391 and 186.

$$\boxed{\phantom{000}} = 391 - 186$$

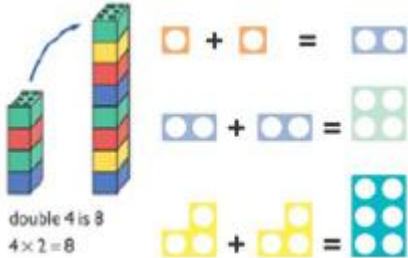

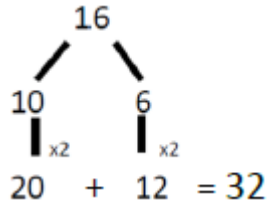


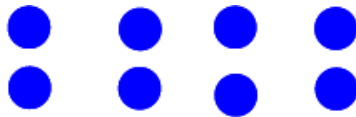
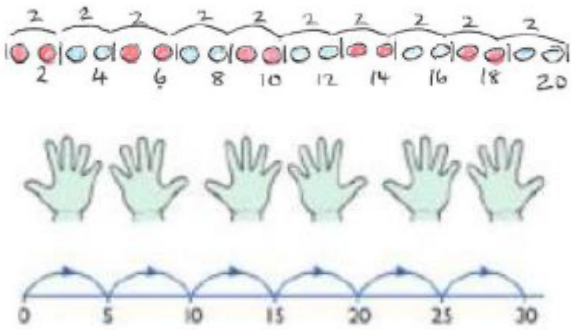
$$\begin{array}{r} 391 \\ - 186 \\ \hline \end{array}$$

What is 186 less than 391?

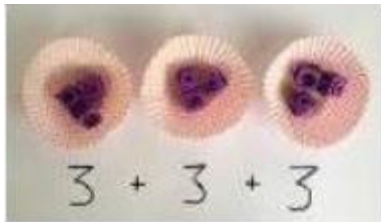
Missing digit calculations

$$\begin{array}{r} 39\boxed{\phantom{0}} \\ - \boxed{\phantom{00}}\boxed{\phantom{00}}6 \\ \hline \boxed{\phantom{00}}05 \end{array}$$

# MULTIPLICATION

Multiplication Year 1	Concrete	Pictorial	Abstract
Doubling	<p>Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling.</p>  <p>double 4 is 8 <math>4 \times 2 = 8</math></p>	<p>Draw pictures to show how to double numbers.</p> <p>Double 4 is 8</p>  <p>Children make representations to show counting in multiples.</p>	<p>Draw pictures to show how to double numbers.</p>  <p>16 10 + 6 <math>20 + 12 = 32</math></p>
Counting in multiples /repeating groups	  <p>Count in multiples supported by concrete objects in equal groups.</p>	 <p>Children make representations to show counting in multiples.</p>  <p>Use a number line or pictures to continue support in counting in multiples.</p>	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 24, 30</p>

Repeated  
addition



Use different  
objects to add  
equal groups.

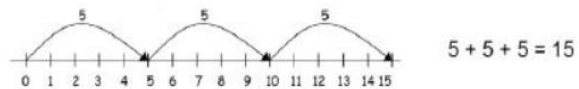
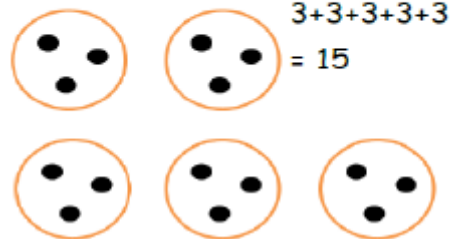


There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?

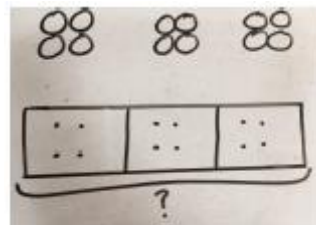
2 add 2 add 2 equals 6



There are 3 sweets in one bag.  
How many sweets are in 5 bags  
altogether?



Use pictorial including number lines to solve  
problems.



Children could  
represent the  
practical resources  
in a picture or use a  
bar model.

Write addition sentences to  
describe objects and pictures.

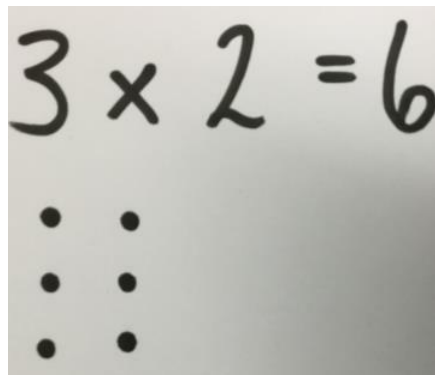


Understanding  
arrays



Create arrays using  
counters/cubes/die  
nes/ base 10 to  
show multiplication  
sentences.

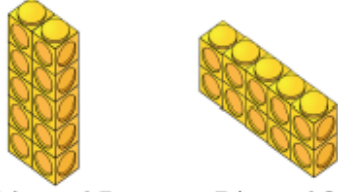
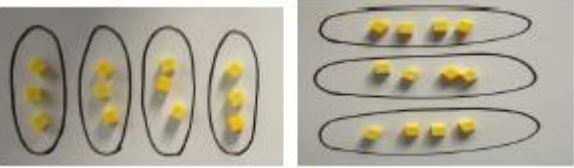
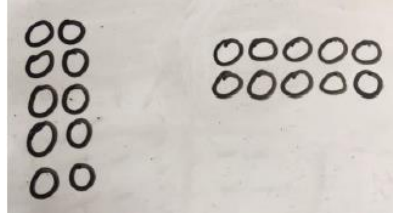
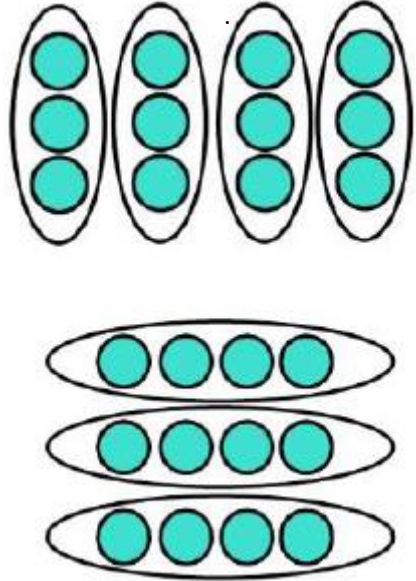

Children to represent the arrays pictorially to  
show their understanding.

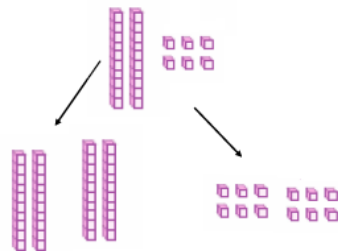
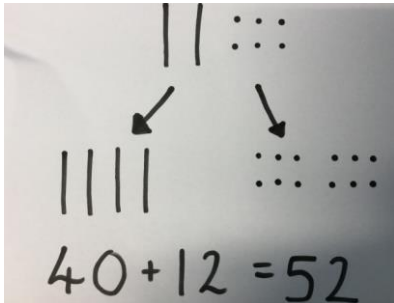
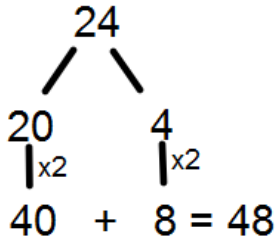
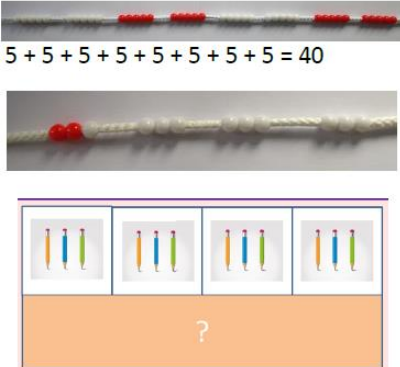

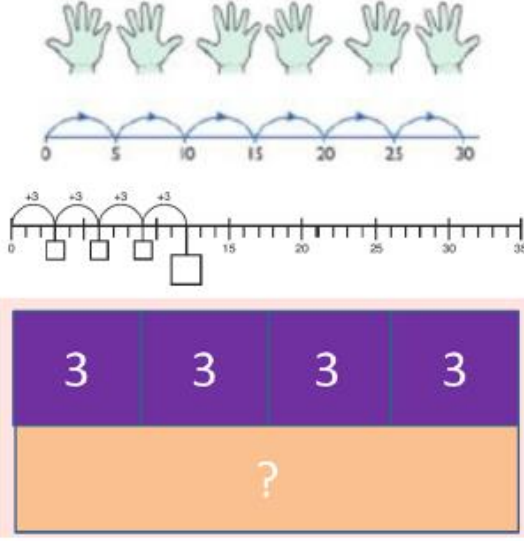


$$3 \times 2 = 6$$

$$2 \times 5 = 10$$

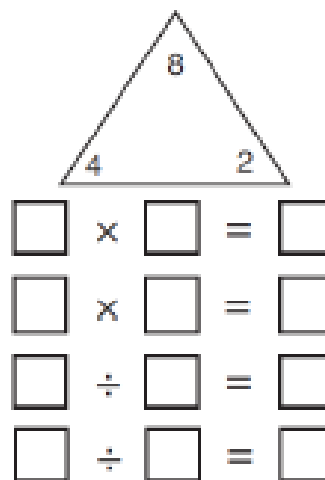


Multiplication Year 2	Concrete	Pictorial	Abstract
<p>Arrays showing commutativity</p>	<p><math>2 \times 5 = 5 \times 2</math></p>  <p>2 lots of 5      5 lots of 2</p> <p>Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of multiplication does not affect the answer.</p> 	 <p>Use representations of arrays to show different calculations and explore commutativity.</p> 	<p><math>12 = 3 \times 4</math></p> <p><math>12 = 4 \times 3</math></p> <p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  <p><math>5 + 5 + 5 = 15</math></p> <p><math>3 + 3 + 3 + 3 + 3 = 15</math></p> <p><math>5 \times 3 = 15</math></p> <p><math>3 \times 5 = 15</math></p>

<p>Doubling</p>	<p>Model doubling using diennes and place value counters. Eg. <math>26 \times 2 =</math></p>  <p><math>40 + 12 = 52</math></p>	<p>Draw pictures and representations to show how to double numbers.</p>  <p><math>40 + 12 = 52</math></p>	<p>Partition a number and then double each part before recombining it back together/</p>  <p><math>40 + 8 = 48</math></p>
<p>Counting in multiples of 2, 3, 4, 5, 10 from 0</p>	<p>Count the groups as children are counting in the multiples. You could use bar models.</p>  	<p>Number lines, counting sticks and bar models should be used to show representation of counting in multiples.</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>0, 2, 4, 6, 8, 10</p> <p>0, 3, 6, 9, 12, 15</p> <p>0, 5, 10, 15, 20, 25, 30</p> <p><math>4 \times 3 = \square</math></p>

Using the inverse

*This should be taught alongside division, so pupils learn how they work alongside each other.*



$$2 \times 4 = 8$$

$$4 \times 2 = 8$$

$$8 \div 2 = 4$$

$$8 \div 4 = 2$$

$$8 = 2 \times 4$$


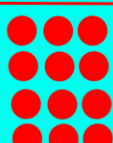
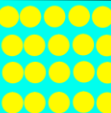
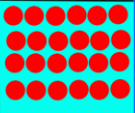


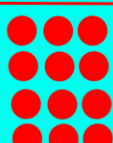
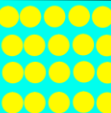
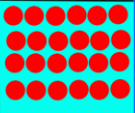
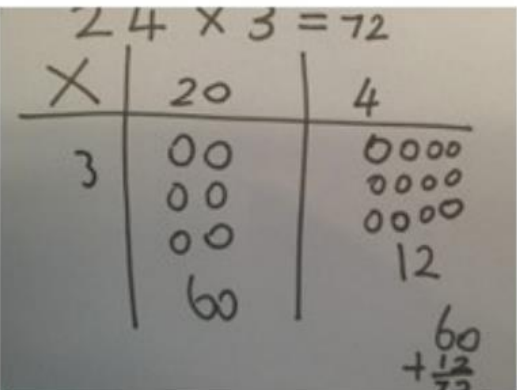


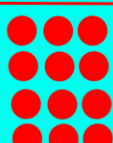
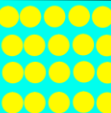
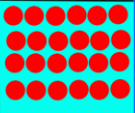
$$8 = 4 \times 2$$

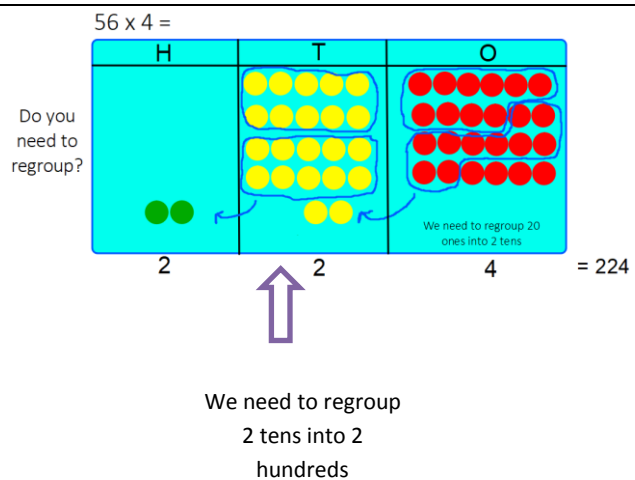
$$2 = 8 \div 4$$

$$4 = 8 \div 2$$

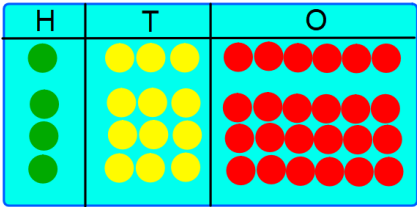
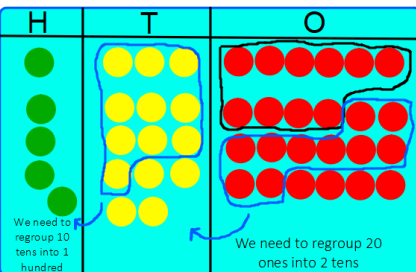
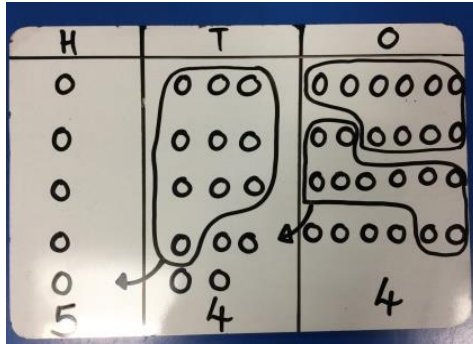
Show all 8 related fact family sentences.

# YEAR 3 MULTIPLICATION

Multiplication Year 3	Concrete	Pictorial	Abstract																		
Grid method	<p>Show the links with arrays first to introduce the grid method.</p> <p>13 x 4 =</p> <table border="1"><tr><td>x</td><td>10</td><td>3</td></tr><tr><td>4</td><td></td><td></td></tr></table> <p>4 rows of 10 4 rows of 3</p> <p>This could also be represented using base ten.</p> <p>Move onto place value counters to show how we are finding groups of a number.</p> <p>We are multiplying by 4 so we need 4 rows.</p> <p>56 x 4 =</p> <table border="1"><tr><td>H</td><td>T</td><td>O</td></tr><tr><td></td><td></td><td></td></tr></table> 	x	10	3	4			H	T	O				<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p> 	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1"><tr><td>x</td><td>30</td><td>5</td></tr><tr><td>7</td><td>210</td><td>35</td></tr></table> <p>210 + 35 = 245</p> <p>If appropriate, NC states that Year 3 should be introduced to a formal written method of short multiplication (TO x O).</p> <p>Expanded Multiplication:</p> <div><p><b>MODEL</b></p><math display="block">\begin{array}{r} 57 \\ 4 \times \\ \hline 28 \quad (4 \times 7) \\ 200 \quad (4 \times 50) \\ \hline 228 \end{array}</math></div>  <div><p><b>MODEL</b></p><math display="block">\begin{array}{r} 57 \\ 4 \times \\ \hline 228 \\ 2 \end{array}</math></div> <p>The children could move onto using short multiplication for TO x O</p>	x	30	5	7	210	35
x	10	3																			
4																					
H	T	O																			
																					
x	30	5																			
7	210	35																			

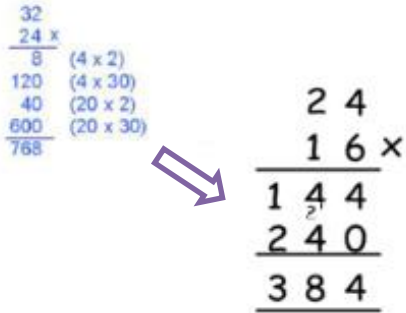


# YEAR 4 YEAR 5 YEAR 6 MULTIPLICATION

Multiplication Year 4, 5 and 6	Concrete	Pictorial	Abstract								
<p>Year 4:</p> <p>Review grid method moving on to 3 digit numbers by 1 digit if children require visual partitioning of digits.</p>	<p>Move onto place value counters to show how we are finding groups of a number.</p> <p><math>136 \times 4 =</math> We are multiplying by 4 so we need 4 rows.</p>  <p><math>136 \times 4 =</math></p> <p>Create 4 groups of 136.</p> <p>Do you need to regroup?</p>  <p>Add up each column, starting with the ones regrouping where needed.</p>		<p><math>234 \times 6 =</math></p> <table border="1"> <tr> <td>X</td><td>200</td><td>30</td><td>4</td></tr> <tr> <td>6</td><td>1200</td><td>180</td><td>24</td></tr> </table> <div> <div>1200</div> <div>180</div> <div>24+</div> <div>1404</div> <div>1</div> </div>	X	200	30	4	6	1200	180	24
X	200	30	4								
6	1200	180	24								



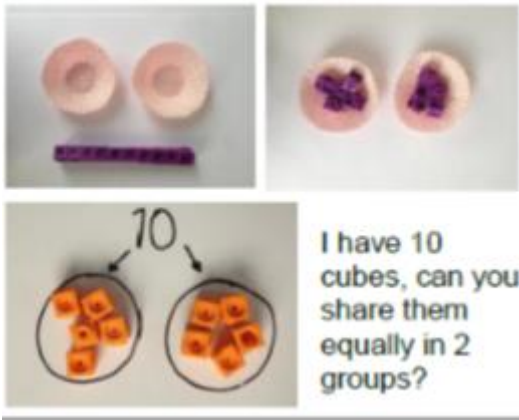
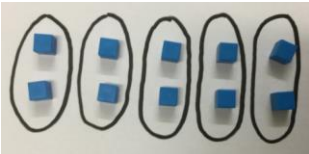
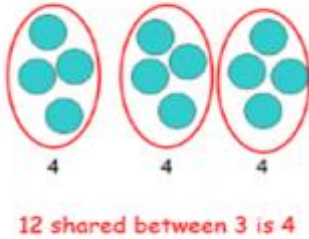
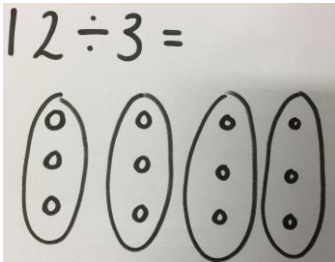
<p>Year 4:</p> <p>Short Multiplication</p> <p>Numbers up to 3 digits.</p>	<p>Children can continue to be supported by place value counters at this stage of multiplication.</p> <p>See above.</p>	<p>The grid method may be used to show how this relates to a formal written method.</p> <table border="1" data-bbox="952 255 1478 383"> <tr> <td>x</td><td>300</td><td>20</td><td>7</td></tr> <tr> <td>4</td><td>1200</td><td>80</td><td>28</td></tr> </table>	x	300	20	7	4	1200	80	28	<div data-bbox="1720 135 2022 513"> <p><b>MODEL</b></p> <math display="block">\begin{array}{r} 327 \\ \times 4 \\ \hline 28 \\ 80 \\ 1200 \\ \hline 1308 \\ 1 \end{array}</math> </div> <div data-bbox="1727 667 2018 962"> <p><b>MODEL</b></p> <math display="block">\begin{array}{r} 327 \\ \times 4 \\ \hline 1308 \\ 12 \end{array}</math> <p>Short Multiplication</p> </div>
x	300	20	7								
4	1200	80	28								
<p>Year 5 and Year 6</p> <p>ThHTO x O</p> <p>ThHTO x TO</p>	<p>Children can continue to be supported by place value counters if necessary.</p>	<p>See above.</p>	<div data-bbox="1688 1011 2047 1374"> <p><b>MODEL</b></p> <math display="block">\begin{array}{r} 2327 \\ \times 5 \\ \hline 11635 \\ 113 \end{array}</math> <p>Short Multiplication</p> </div>								


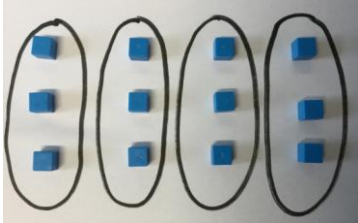
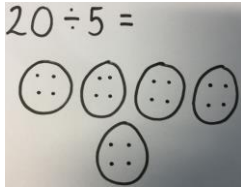
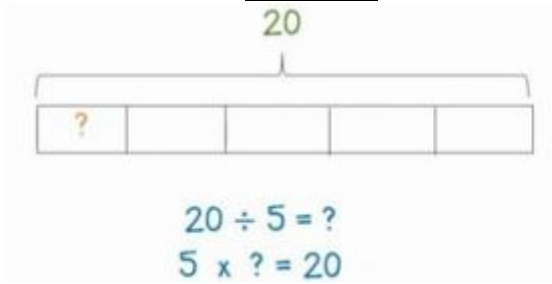
		<p>When children start to multiply 3 digit by 3 digit and 4 digit by 2 digit etc., they should be confident with the abstract.</p>	<p>In year 5, children are expected to be able to use long multiplication for two digit numbers.</p> <p>Start with long multiplication, reminding the children about lining up their numbers clearly in columns.</p> <p>If it helps, children can write out what they are solving next to their answer.</p> <div style="text-align: center;">  </div> <p>In Year 6, children are expected to multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.</p>
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			$  \begin{array}{r}  2741 \\  \times 66 \\  \hline  16446 \\  164460 \\  \hline  180906 \\  \begin{array}{cc} 1 & 1 \end{array}  \end{array}  $
Year 6:			<p>Children are required to multiply one-digit numbers with up to two decimal places by whole numbers (NC-Fractions including decimals)</p> <p>Remind children that the single digit belongs in the units column. Line up the decimal points in the question and in the answer.</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> <math display="block">  \begin{array}{r}  3.19 \\  \times 8 \\  \hline  25.52  \end{array}  </math> </div> <div style="text-align: center;"> <math display="block">  \begin{array}{r}  2.41 \\  \times 6 \\  \hline  14.46 \\  2  \end{array}  </math> </div> </div>

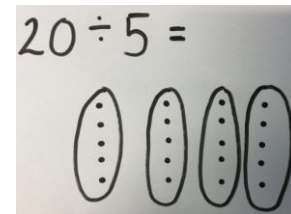
Conceptual variation; different ways to ask children to solve  $6 \times 23$

<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <div style="display: flex; justify-content: space-between; border-bottom: 1px solid black; margin-bottom: 5px;"> <span>23</span><span>23</span><span>23</span><span>23</span><span>23</span><span>23</span> </div> <div style="text-align: center; height: 40px; border-top: 1px solid black;">?</div> </div>	<p>Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?</p> <p>With the counters, prove that <math>6 \times 23 = 138</math></p>	<p>Find the product of 6 and 23</p> <p><math>6 \times 23 =</math></p> <div style="border: 1px dashed blue; padding: 5px; display: inline-block; margin-bottom: 10px;"> <math>\square = 6 \times 23</math> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> <math display="block">  \begin{array}{r}  6 \\  \times 23 \\  \hline  \end{array}  </math> </div> <div style="text-align: center;"> <math display="block">  \begin{array}{r}  23 \\  \times 6 \\  \hline  \end{array}  </math> </div> </div>	<p>What is the calculation? What is the product?</p> <table border="1" style="width: 100%; text-align: center; border-collapse: collapse;"> <tr style="background-color: #f8d7da;"> <th>100s</th> <th>10s</th> <th>1s</th> </tr> <tr> <td style="height: 100px;"></td> <td style="height: 100px;"> <div style="display: flex; flex-direction: column; align-items: center;"> <span>6</span><span>2</span><span>3</span> </div> </td> <td style="height: 100px;"> <div style="display: flex; flex-direction: column; align-items: center;"> <span>6</span><span>2</span><span>3</span> </div> </td> </tr> </table>	100s	10s	1s		<div style="display: flex; flex-direction: column; align-items: center;"> <span>6</span><span>2</span><span>3</span> </div>	<div style="display: flex; flex-direction: column; align-items: center;"> <span>6</span><span>2</span><span>3</span> </div>
100s	10s	1s							
	<div style="display: flex; flex-direction: column; align-items: center;"> <span>6</span><span>2</span><span>3</span> </div>	<div style="display: flex; flex-direction: column; align-items: center;"> <span>6</span><span>2</span><span>3</span> </div>							

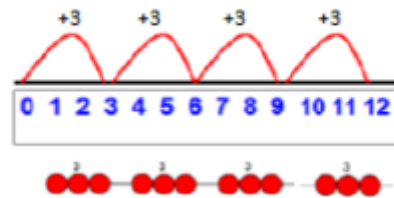
Division Year 1	Concrete	Pictorial	Abstract
Sharing objects into group Division as grouping	<p><u>Sharing:</u></p>  <p>I have 10 cubes, can you share them equally in 2 groups?</p> <p><u>Grouping:</u></p> <p>I have 10 cubes. Can you group them into 2s?</p> 	<p><u>Sharing:</u></p>  <p>12 shared between 3 is 4</p> <p><u>Grouping:</u></p> 	<p><u>Sharing:</u></p> <p>Share 9 buns between three people.</p> $9 \div 3 = 3$ <p><u>Grouping:</u></p> <p>Divide 15 into groups of 5. How many groups are there?</p>

Division Year 2	Concrete	Pictorial	Abstract
Division as sharing and grouping	<p><u>Sharing:</u></p>  $12 \div 3 = 4$ <p><u>Grouping:</u></p>  $12 \div 3 = 4$	<p><u>Sharing:</u></p>  <p><u>Bar Model:</u></p>  <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p>	<p><u>Sharing:</u></p> $28 \div 7 = 4$ <p>Divide 28 into 7 groups. How many are in each group?</p> <p><u>Grouping:</u></p> <p>Divide 28 into groups of 7. How many groups are there?</p>

Grouping:


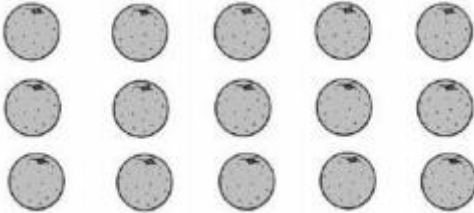


Use number lines for grouping



$$12 \div 3 = 4$$

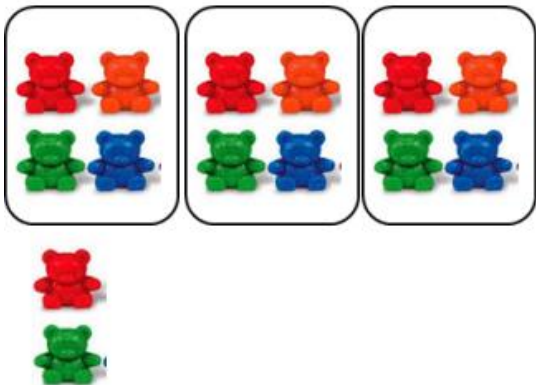


Division Year 3	Concrete	Pictorial	Abstract
Division with arrays	 <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>Eg <math>15 \div 3 = 5</math>    <math>5 \times 3 = 15</math>  <math>15 \div 5 = 3</math>    <math>3 \times 5 = 15</math></p>	<p>Draw an array and use lines to split the array into groups to make multiplication and division sentences</p> 	<p>Find the inverse of multiplication and division sentences by creating eight linking number sentences.</p> <p><math>7 \times 4 = 28</math>  <math>4 \times 7 = 28</math>  <math>28 \div 7 = 4</math>  <math>28 \div 4 = 7</math>  <math>28 = 7 \times 4</math>  <math>28 = 4 \times 7</math>  <math>4 = 28 \div 7</math>  <math>7 = 28 \div 4</math></p>

Division  
using  
remainders

$14 \div 3 =$

Divide objects between groups and see how much is left over



2d ÷ 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used.

13 ÷ 4

Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.

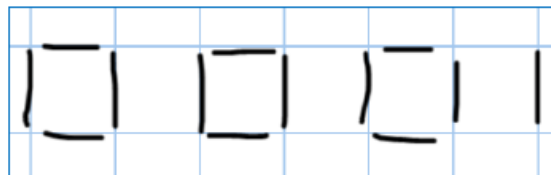


There are 3 whole squares, with 1 left over.

Draw dots and group them to divide an amount and clearly show a remainder.



Children to represent the lollipop sticks pictorially.



There are 3 whole squares, with 1 left over.

Complete written divisions and show the remainder using  $r$ .

$$\begin{array}{ccccccc} 29 & \div & 8 & = & 3 & \text{REMAINDER} & 5 \\ \uparrow & & \uparrow & & \uparrow & & \uparrow \\ \text{dividend} & & \text{divisor} & & \text{quotient} & & \text{remainder} \end{array}$$

$13 \div 4 = 3$  remainder 1

Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

'3 groups of 4, with 1 left over'

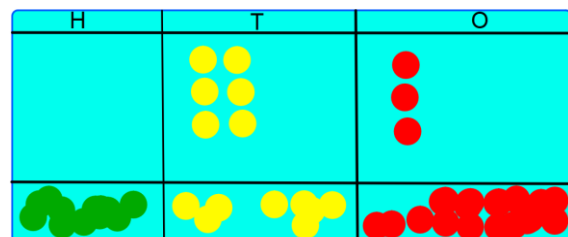
Introducing  
Short  
division (no  
remainders)

Grouping  
with place  
value  
counters

(no  
regrouping)

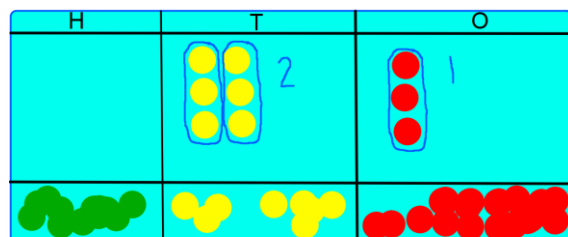
Use place value counters to introduce dividing  
using short division. Where appropriate children to  
record their response in the short division method.

$$63 \div 3 =$$



How many groups  
of 3 in the tens?

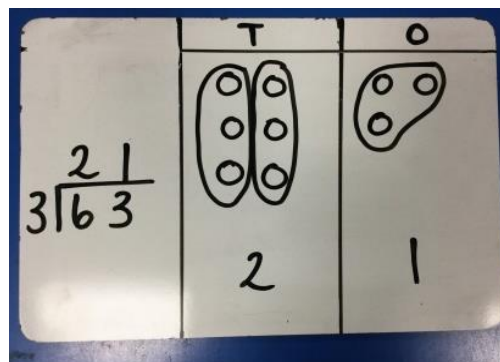
How many groups of  
3 in the units/ones?



How many groups  
of 3 in the tens?

How many groups of  
3 in the units/ones?

$$\begin{array}{r} 21 \\ 3 \overline{) 63} \end{array}$$



$$\begin{array}{r} 21 \\ 4 \overline{) 84} \end{array}$$

$$\begin{array}{r} 43 \\ 2 \overline{) 86} \end{array}$$

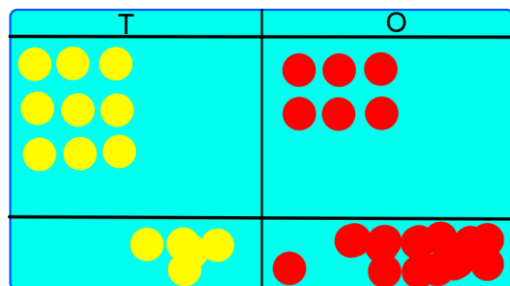
$$\begin{array}{r} 32 \\ 3 \overline{) 96} \end{array}$$

Introducing  
Short  
division

Grouping  
with place  
value  
counters

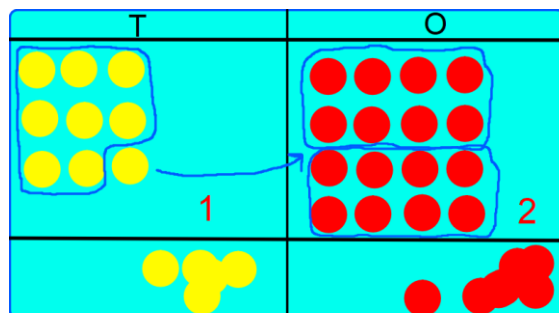
(regrouping)

$$96 \div 8 =$$



How many groups of 8 in the tens?

How many groups of 8 in the units/ones?

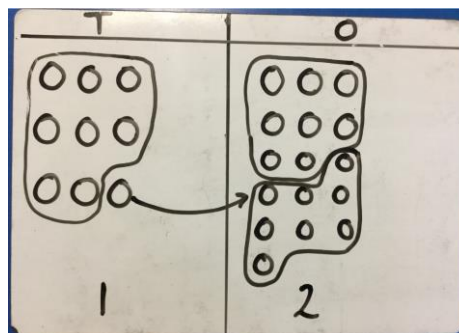


How many groups of 8 in the tens?

How many groups of 8 in the units/ones?

We need to exchange  
1 ten from the tens  
into 10 ones.

$$8 \overline{) 96} \begin{matrix} 12 \\ 12 \end{matrix}$$



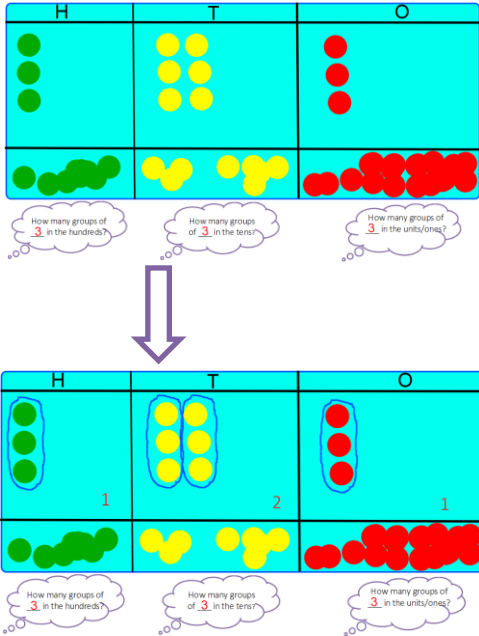
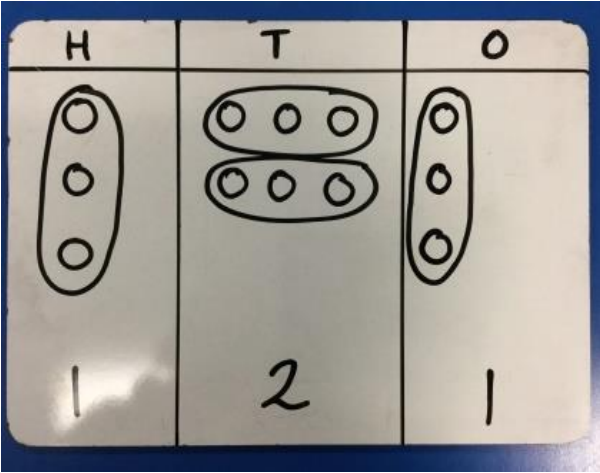
Encourage the children to record their answer in  
the short division format.

$$3 \overline{) 42} \begin{matrix} 14 \\ 12 \end{matrix}$$

$$4 \overline{) 56} \begin{matrix} 14 \\ 16 \end{matrix}$$

$$6 \overline{) 96} \begin{matrix} 16 \\ 36 \end{matrix}$$

# YEAR 4 YEAR 5 YEAR 6 DIVISION

Division Year 4, 5 and 6	Concrete	Pictorial	Abstract
<p>Divide at least 3 digit numbers by 1 digit.</p> <p>(no regrouping)</p> <p>Short Division</p>	<p>Use place value counters to introduce dividing 3 digit numbers using short division. Alongside using the place value counters, children need to record their answer in short division</p> <p><math>363 \div 3 =</math></p>  <p><math>363 \div 3 = 121</math></p> $\begin{array}{r} 121 \\ 3 \overline{) 363} \end{array}$	<p>Encourage the children to record their answer in the short division format.</p> 	$\begin{array}{r} 211 \\ 4 \overline{) 844} \end{array}$ $\begin{array}{r} 321 \\ 2 \overline{) 642} \end{array}$ $\begin{array}{r} 322 \\ 3 \overline{) 966} \end{array}$

Divide at least 3 digit numbers by 1 digit. (regrouping)

Short Division

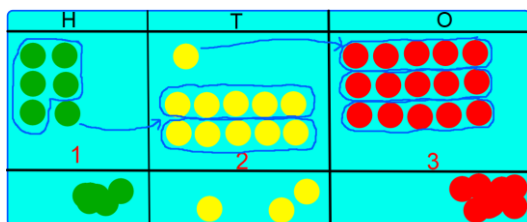
$$615 \div 5 =$$



How many groups of 5 in the hundreds?

How many groups of 5 in the tens?

How many groups of 5 in the units/ones?



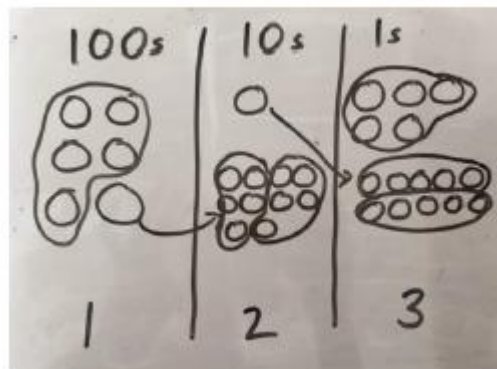
How many groups of 5 in the hundreds?

How many groups of 5 in the tens?

How many groups of 5 in the units/ones?

Encourage the children to record their short division calculation alongside their use of place value counters.

Represent the place value counters pictorially.



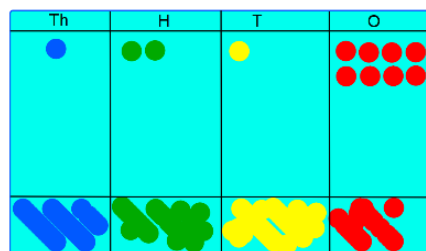
Encourage the children to record their answer in the short division format.

Children to the calculation using the short division scaffold.

$$\begin{array}{r} 123 \\ 5 \overline{) 615} \\ \underline{5} \phantom{00} \\ 11 \phantom{0} \\ \underline{10} \phantom{0} \\ 15 \\ \underline{15} \\ 0 \end{array}$$

Year 6

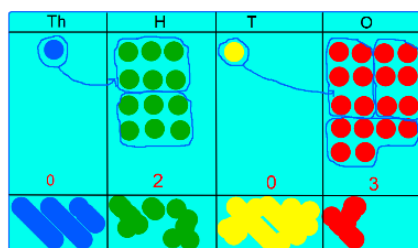
Divide at least 4 digit numbers by 1 digit.  
(regrouping)



How many groups of 6 in the hundreds?

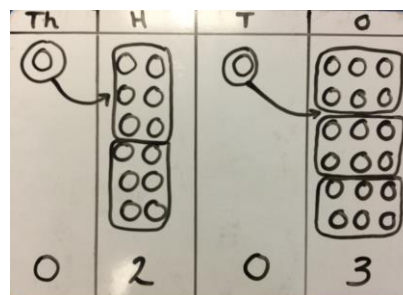
How many groups of 6 in the tens?

How many groups of 6 in the units/ones?



$$\begin{array}{r} 0203 \\ 6 \overline{) 1218} \end{array}$$

Short Division



Encourage the children to record their answer in the short division format.

$$\begin{array}{r} 0203 \\ 6 \overline{) 1218} \end{array}$$

$$\begin{array}{r} 45r1 \\ 11 \overline{) 496} \end{array}$$

Answer  $45 \frac{1}{11}$

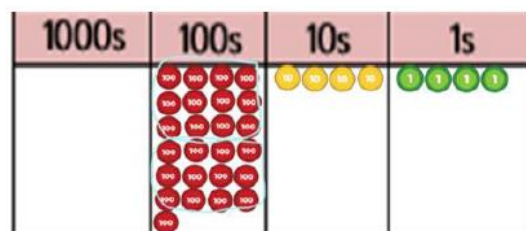


Long  
division  
  
(Year 6)

Long division using place value counters  
 $2544 \div 12$

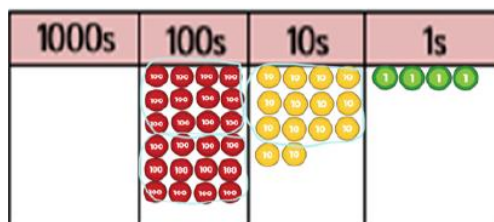


We can't group 2 thousands into groups of 12 so will exchange them.



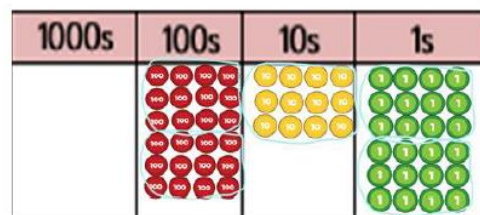
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \phantom{0} \\ 1 \phantom{0} \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \phantom{0} \\ 14 \phantom{0} \\ \underline{12} \phantom{0} \\ 2 \phantom{0} \end{array}$$

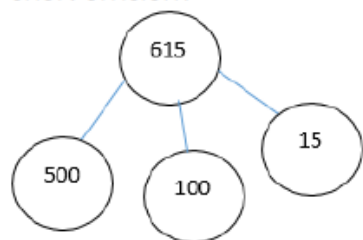


After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \phantom{0} \\ 14 \phantom{0} \\ \underline{12} \phantom{0} \\ 24 \phantom{0} \\ \underline{24} \phantom{0} \\ 0 \end{array}$$

# Conceptual variation; different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{)615}$$

$$615 \div 5 =$$

$$\square = 615 \div 5$$

What is the calculation?  
What is the answer?

